



Metric Based Comparison

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Agenda

- Overview of Comparison Techniques
- Examples of distance and statistical approaches
- ORA example



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Comparison Techniques

- Visualization
- Distance
- Statistical approaches
 - Summary statistics
 - Confidence intervals for measures
 - Node level statistics
 - QAP Models & MrQAP
 - Discussed in Day 3 Talks

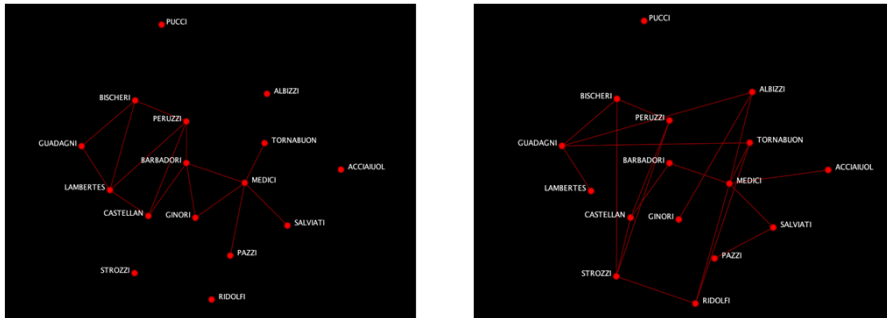
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Comparison by Visualization

- Padgett Medici Banking Ties
- Padgett Medici Marriage Ties



The figure displays two network graphs side-by-side, comparing banking ties (left) and marriage ties (right) among the Padgett Medici family. Both graphs use red nodes and edges on a black background. The nodes represent family members: PUCCI, BISCHERI, PERUZZI, ALBIZZI, GUADAGNI, BARRADORI, TORNABUON, ACCIAIUOL, LAMBERTES, CASTELLAN, GINORI, MEDICI, SALVIATI, STROZZI, PAZZI, and RIDOLFI. The marriage ties graph shows a more dense network of connections compared to the banking ties graph.

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See Differences by Utilizing ORA Tiles

- Banking Ties
- Nodes sized by betweenness and colored by degree
- Marriage Ties
- Nodes sized by betweenness and colored by degree

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Overlaying Networks in ORA

- Red ties are banking
- Blue ties are marriage
- Nodes sized by betweenness and colored by degree

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Comparison Technique: Distance

- If a network is treated like a string, then the distance between two strings can be calculated by a number of metrics
- Most common distance metrics for networks
 - Hamming
 - Binary
 - Euclidean
 - Non-binary

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First Distance Technique: Hamming Distance

- The Hamming Distance of two networks (with the same nodeset) is the number of times a link exists in one network but not the other.
- In ORA, it's found in the Key Entities Ranking report.
- It's reported as a percentage:
 - Hamming Difference = $100 * (\text{Max possible distance} - \text{Hamming}) / \text{Max possible distance}$

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Hamming Distance Example

- Take the following two networks and convert them to binary strings:

Network 1

	A	B	C	D	E
A	0	1	1	1	0
B	1	0	0	0	0
C	1	0	0	0	1
D	0	0	0	0	1
E	1	1	1	1	0

Network 2

	A	B	C	D	E
A	0	0	0	1	0
B	1	0	0	0	0
C	1	0	0	0	1
D	0	0	1	0	0
E	1	1	1	0	0

0111010000100010000111110
0001010000100010010011100

- There are 5 differences between the two strings. The unnormalized hamming distance is **5**.
- Since there are 20 possible links, the normalized hamming distance is $5/20 = .2$ or 20%.

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Variation of Hamming Distance: Damerau-Levenshtein Distance

- Damerau-Levenshtein Distance is the distance found by counting the minimum number of operations needed to transform one string into the other.
- Operations include:
 - Insertion
 - Deletion
 - Substitution of a single character
 - Transposition of two adjacent characters

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Finding Distances Using Non-Binary Data

- Euclidean Distance
 - Physical interpretation of distance
 - Square root of sum of squares of differences between cells

$$d(x, y) = \sqrt{\sum_{i=1}^n (x_i - y_i)^2}$$

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Analyzing Social Networks Using Statistics

- We can use statistical analysis to estimate how precise a given description is when comparing groups.
- Assumptions:
 - Descriptive statistics are fine
 - Standard error is needed
 - Ideally want to not have strong assumptions about how the network was generated


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Comparing Networks: More Florentine Families

Social Density, Weighted	
Density of the agent x agent network(s) taking into account link weights.	
PADGM	0.167
PADGB	0.125
Social Fragmentation	
The fragmentation (amount of disconnectivity of nodes) of the agent x agent network(s).	
PADGM	0.125
PADGB	0.542
Avg Communication Speed	
The average speed with which any two (reachable) nodes can interact. This is the inverse of the average shortest path length between node pairs. If no node is reachable from another node, then Minimum Speed is zero.	
PADGM	0.402
PADGB	0.420
Communication Network Diameter	
The maximum shortest path length between any two nodes in a unimodal network. If there exists a node that is not reachable from another node, then the diameter is technically infinite. In this case, the Diameter returned is $V*N$ where V is the maximum link value in the network.	
PADGM	16
PADGB	16

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Distributions of Node Level Metrics

Total Degree Centrality

Individuals or organizations who are 'in the know' are those who are linked to many others and so, by virtue of their position have access to the ideas, thoughts, beliefs of many others. Individuals who are 'in the know' are identified by degree centrality in the relevant social network. Those who are ranked high on this metric have more connections to others in the same network. The scientific name of this measure is total degree centrality and it is calculated on the agent by agent matrices.

If the node of interest has a higher than normal value (greater than 1 standard deviation(s) above the mean) the row is colored red. The row is green if the node is within 1 standard deviation of the mean. Finally, the row is colored blue if the node has a lower than normal value (less than one standard deviation(s) below the mean).

Input network: PADGB (size: 16, density: 0.125)


Show (10 of 16) entries Search:

Rank	Agent	Value	Unscaled	Context*
1	MEDICI	0.333	5	2.520
2	BARBADORI	0.267	4	1.713
3	LAMBERTES	0.267	4	1.713
4	PERUZZI	0.267	4	1.713
5	BISCHERI	0.200	3	0.907
6	CASTELLAN	0.200	3	0.907
7	GINORI	0.133	2	0.101
8	GUADAGNI	0.133	2	0.101
9	PAZZI	0.067	1	-0.706
10	SALVIATI	0.067	1	-0.706

Showing 1 to 10 of 16 entries Previous 1 2 Next

* Number of standard deviations from the mean of a random network of the same size and density

Value statistics					
Min:	0	Mean:	0.125	Mean in random network:	0.125
Max:	0.333	Std.dev:	0.113	Std.dev in random network:	0.083
				Lower quartile:	0
				Median:	0.100
				Upper quartile:	0.233

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Distributions of Node Level Statistics

- **Banking Network**
 - Mean = .0724
 - Std. Deviation = .08898
- **Marriage Network**
 - Mean = .1467
 - Std. Deviation = .1133

Centrality, Betweenness: PADGB

Centrality, Betweenness: PADGM

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Comparing Node Level Statistics

Total Degree Centrality

Individuals or organizations who are 'in the know' are those who are linked to many others and so, by virtue of their position have access to the ideas, thoughts, beliefs of many others. Individuals who are 'in the know' are identified by degree centrality in the relevant social network. Those who are ranked high on this metrics have more connections to others in the same network. The scientific name of this measure is total degree centrality and it is calculated on the agent by agent matrices.

If the node of interest has a higher than normal value (greater than 1 standard deviation(s) above the mean) the row is colored **red**. The row is green if the node is within 1 standard deviation of the mean. Finally, the row is colored **blue** if the node has a lower than normal value (less than one standard deviation(s) below the mean).

Showing 1 to 10 of 16 entries Search:

Rank #	PADGM					PADGB				
	Agent	Value	% DMT to Case 2	Unscaled	% DMT to Case 2	Agent	Value	% DMT from Case 1	Unscaled	% DMT from Case 1
1	MEDICI	0.400	+16.67%	6	+16.67%	MEDICI	0.333	-20.00%	5	-20%
2	GUADAGNI	0.267	+50%	4	+50%	BARBADORI	0.267	+50%	4	+50%
3	STROZZI	0.267	+100%	4	+100%	LAMBERTES	0.267	+75%	4	+75%
4	ALBIZZI	0.200	+100%	3	+100%	PERUZZI	0.267	+25.00%	4	+25%
5	BISCHERI	0.200	+0%	3	+0%	BISCHERI	0.200	+0%	3	+0%
6	CASTELLAN	0.200	+0%	3	+0%	CASTELLAN	0.200	+0%	3	+0%
7	PERUZZI	0.200	-33.33%	3	-33.33%	GINORI	0.133	+50%	2	+50%
8	RIDOLFI	0.200	+100%	3	+100%	GUADAGNI	0.133	-100%	2	-100%
9	TORNABUON	0.200	+66.67%	3	+66.67%	PAZZI	0.067	+0%	1	+0%
10	BARBADORI	0.133	-100%	2	-100%	SALVIATI	0.067	-100%	1	-100%

Showing 1 to 10 of 16 entries Previous 1 2 Next

padggt value statistics				padggb value statistics			
Min:	0	Lower quartile:	0.067	Min:	0	Lower quartile:	0
Max:	0.400	Median:	0.200	Max:	0.333	Median:	0.100
Mean:	0.167	Upper quartile:	0.200	Mean:	0.125	Upper quartile:	0.233
Std.dev:	0.097			Std.dev:	0.113		

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Percentage Difference

- Given two networks, A and B .
- $Percentage\ Difference = 100 * \frac{A-B}{A}$
- If B is greater than A , the result will be negative
- Any two metrics can be compared this way

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When Key Actors are Removed from the Network

- Redundancy decreases
- Intellectual property is removed or decreased
- Performance, adaptability, and information diffusion are altered
- Cellular networks can withstand high levels of turnover

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Violated Assumptions when Comparing Two Networks

- Networks have row/column dependencies
- Each entry is a dyad, and dyads *aren't* independent
- This violates the assumption of standard statistics
- Violation of the assumption leads to explanatory power attributable to interdependence between nodes that is falsely attributes to the covariates
- Solutions would either require dummy row/column data or estimating a covariance matrix...
 - Empirical standard errors can estimate errors by using permutations of the dataset (QAP / MRQAP)

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Demonstration in ORA

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