



Synthetic Networks

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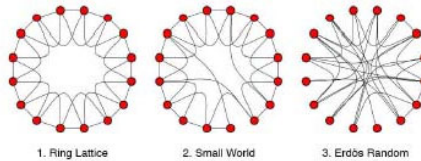
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Overview

- Network researchers have identified some “stylized” network structures
- Stylized networks in ORA

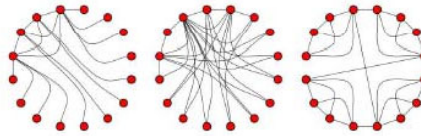
- Ring Lattice
- Small world
- Erdos-Renyi
- Core-Periph
- Scale Free
- Cellular



1. Ring Lattice

2. Small World

3. Erdos Random



4. Core Periphery

5. Scale Free

6. Cellular



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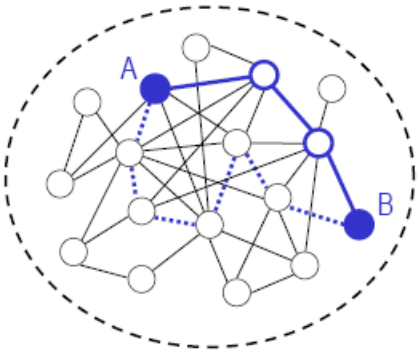
Why Synthetic Networks?

- Synthetic networks generated using random network generators
 - Easy to obtain
 - Can vary parameters when running experiments
- **Real-life (empirical) networks often compared to stylized networks**
- Have characteristics of real social networks
 - Clustering
 - Degree distribution

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Structural metrics: Average path length



The path length between A and B is 3

- the *path length* between two nodes A and B is the smallest number of edges connecting them:
$$l(A, B) = \min l(A, A_1, \dots, A_n, B)$$
- the *average path length* of a network over all pairs of N nodes is
$$L = \langle l(A, B) \rangle$$
$$= \frac{2}{N(N-1)} \sum_{A,B} l(A, B)$$
- the *network diameter* is the maximal path length between two nodes:
$$D = \max l(A, B)$$
- property: $1 \leq L \leq D \leq N-1$

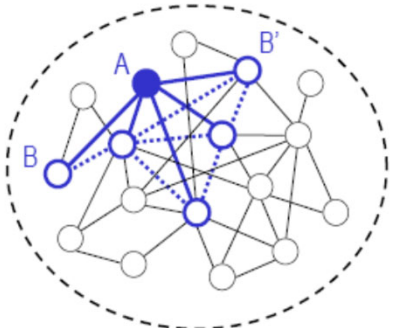
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Structural Metrics: Clustering coefficient



The clustering coefficient of A is 0.6

- the *neighborhood* of a node A is the set of k_A nodes at distance 1 from A
- given the number of *pairs* of neighbors:
$$F_A = \sum_{B, B'} 1 = k_A(k_A - 1) / 2$$
- and the number of pairs of neighbors that are also *connected* to each other:
$$E_A = \sum_{B \leftrightarrow B'} 1$$
- the *clustering coefficient* of A is
$$C_A = E_A / F_A \leq 1$$
- and the *network clustering coefficient*:
$$\langle C \rangle = 1/N \sum_A C_A \leq 1$$

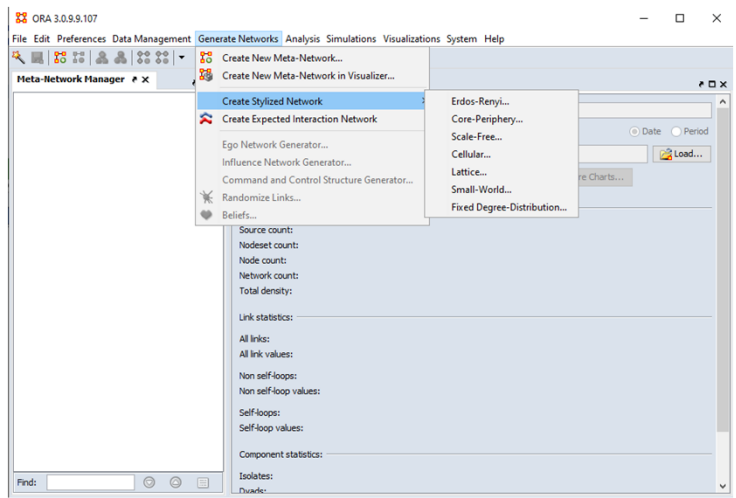
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ORA – Getting Started



ORA 3.0.0.9.107

File Edit Preferences Data Management Generate Networks Analysis Simulations Visualizations System Help

Meta-Network Manager x

- Create New Meta-Network...
- Create New Meta-Network in Visualizer...
- Create Stylized Network
- Create Expected Interaction Network
- Ego Network Generator...
- Influence Network Generator...
- Command and Control Structure Generator...
- Randomize Links...
- Beliefs...

- Erdos-Renyi...
- Core-Periphery...
- Scale-Free...
- Cellular...
- Lattice...
- Small-World...
- Fixed Degree-Distribution...

Source count:
Nodeset count:
Node count:
Network count:
Total density:

Link statistics:
All links:
All link values:
Non self-loops:
Non self-loop values:
Self-loops:
Self-loop values:

Component statistics:
Isolates:
Duads:

Find: []

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Erdos-Renyi (Random)

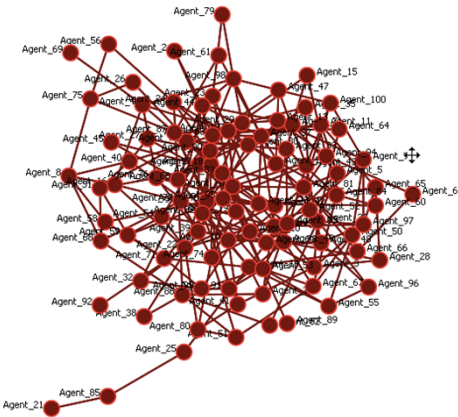
- Purely random
- Most common form studied
- Properties
 - Simplest network
 - Short distances
 - No local structure (clustering) (till a threshold)
 - Very different than real-world networks
 - Rich theory, explains small diameter and giant component

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Erdos-Renyi

- Purely random
- Most common form studied
- Properties
 - Short distances
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 - Very different than real-world networks



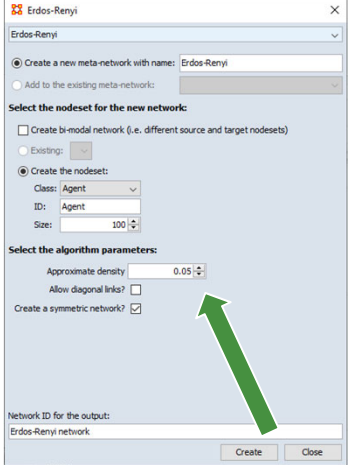
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Erdos-Renyi

- Input: approx. density of resulting network



Approximate density of resulting network

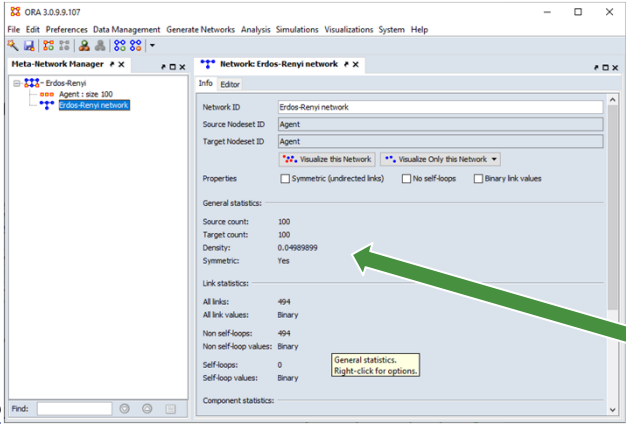
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Erdos-Renyi

- Input: approximate density of resulting network



Approximate density of resulting network

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Erdos-Renyi

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Erdos-Renyi – bi-modal

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Erdos-Renyi – bi-modal

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Erdos-Renyi Notes

- Most common form studied
- Statistical tests to decide if your network is random
- Easy to generate
- Good mathematical properties
- Very different than real world networks

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Ring Lattice

- Nodes laid out in a circle and connected to their K-closest neighbors
- Properties
 - High clustering
 - High average path length

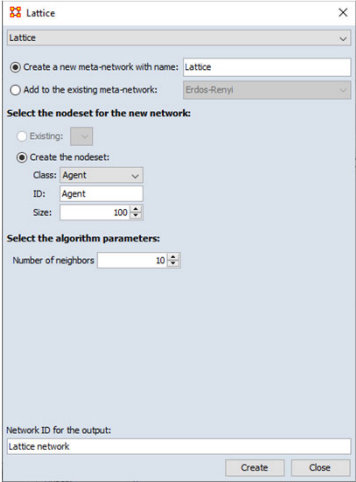
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Ring Lattice

- Number of agents: network size
- Number of neighbors: number of neighbors each node is connected to



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Ring Lattice

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Small World

- Three Steps
 - Begin with a lattice, e.g. 2D grid with k-nearest neighbors connected
 - Randomly remove connections
 - Randomly add long-distance connections
- Properties
 - High local structure (clustering)
 - Short maximum distances

Examples: Telephone call graphs, electric power grids

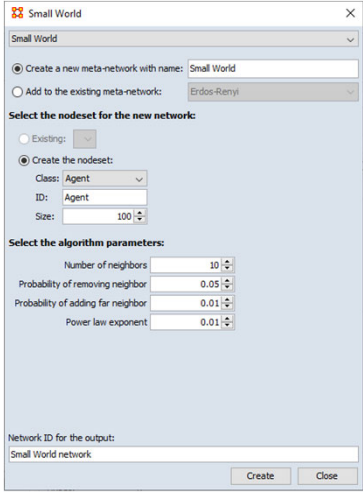
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Small World

- Number of neighbors: Dimensionality of embedding space/lattice. Alternatively, average degree.
- Probability of removing a neighbor: Remove any local structure?
- Probability of adding a neighbor: Add long-range connections?
- Power law exponent: How much should long-range connections ignore local structure? How far should they be?



Small World dialog box settings:

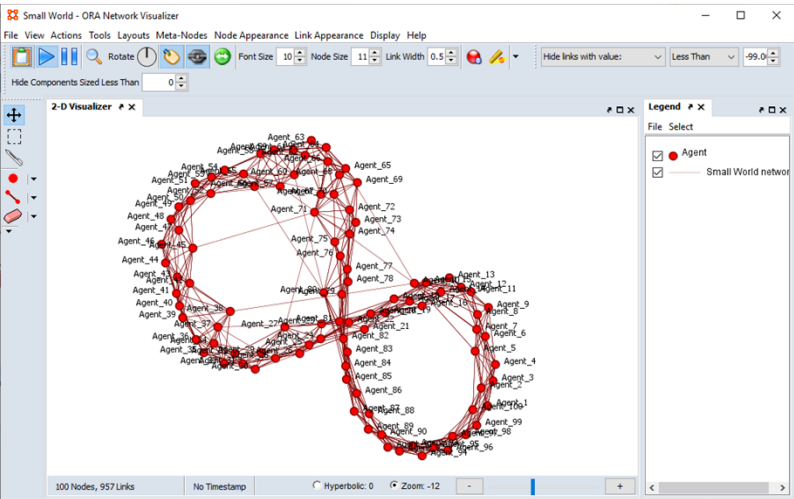
- Small World
- Create a new meta-network with name: Small World
- Add to the existing meta-network: Erdos-Renyi
- Select the nodeset for the new network:
 - Existing:
 - Create the nodeset:
 - Class: Agent
 - ID: Agent
 - Size: 100
- Select the algorithm parameters:
 - Number of neighbors: 10
 - Probability of removing neighbor: 0.05
 - Probability of adding far neighbor: 0.01
 - Power law exponent: 0.01
- Network ID for the output: Small World network
- Create Close

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Small World



Small World - ORA Network Visualizer

File View Actions Tools Layouts Meta-Nodes Node Appearance Link Appearance Display Help

Hide Components Sized Less Than 0

2-D Visualizer

Legend

- Agent
- Small World network

100 Nodes, 957 Links No Timestamp Hyperbolic: 0 Zoom: -12

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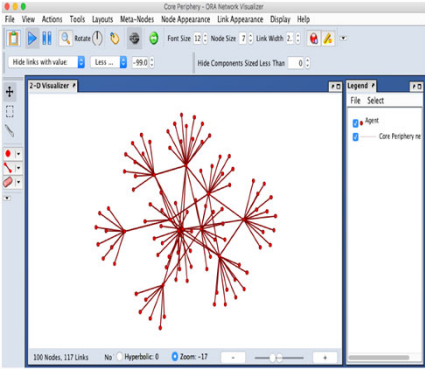


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Core-Periphery

- Two kinds of nodes
 - Core of interconnected nodes
 - Periphery of pendants with single connection to Core
- Properties
 - Short distances
 - Some local structure (core vs non-core)

Examples: Observed trade flows, diplomatic ties among countries



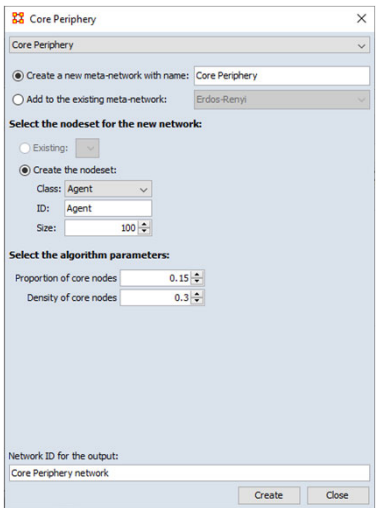
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Core-Periphery

- Proportion of core nodes
- Density of core nodes: How dense should the within-core network be?



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Core-Periphery

100 Nodes, 138 Links

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Cellular

- Small number of tight clusters with few links between clusters
- Properties
 - Large distances
 - High local structure (clustering)

Examples: Terrorist networks

100 Nodes, 434 Links

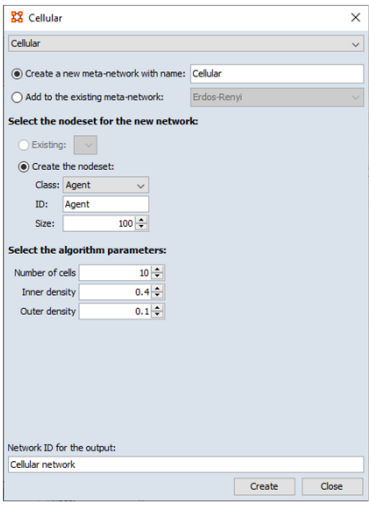
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Cellular

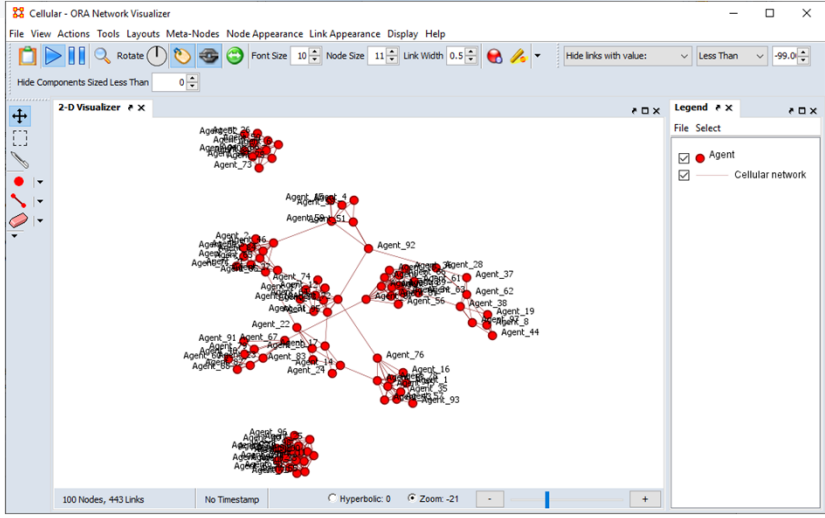
- Number of cells: How many cells?
- Inner Density: how dense should the network within each cell be?
- Outer Density: how dense should connections between cells be?



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Cellular

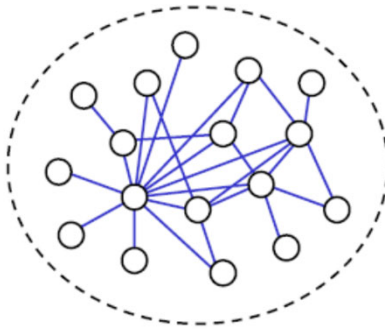


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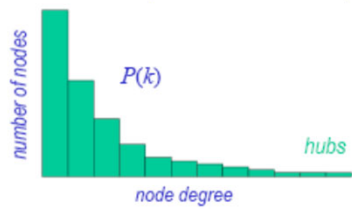
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Scale-Free Networks – Degree Distribution



A schematic scale-free network

- in a *scale-free network* the degree distribution follows a **POWER-LAW**:
$$P(k) \sim k^{-\gamma}$$
- there exists a small number of highly connected nodes, called *hubs* (tail of the distribution)
- the great majority of nodes have few connections (head of the distribution)



number of nodes

$P(k)$

node degree

hubs

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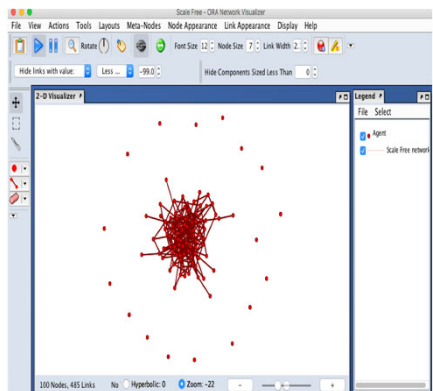
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Scale-Free

- Process
 - Some initial network
 - New nodes more likely to connect to existing nodes with high degree
- Properties
 - Short distances
 - No local structure



Scale Free - CNA Network Visualizer

File View Actions Tools Layouts Meta-Nodes Node Appearance Link Appearance Display Help

Hide links with value: Less -99.0 Hide Components Sized Less Than 0

2-D Visualizer

Legend

- 4897
- Scale Free network

100 Nodes, 485 Links No Hyperbolic: 0 Zoom: -22

Examples: Social networks, Computer networks (WWW)

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Scale-Free

- How likely is it for new nodes to connect to the core?
- Initial node count: How big should the initial network be?
- Initial density: How dense should the initial network be?

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Scale-Free

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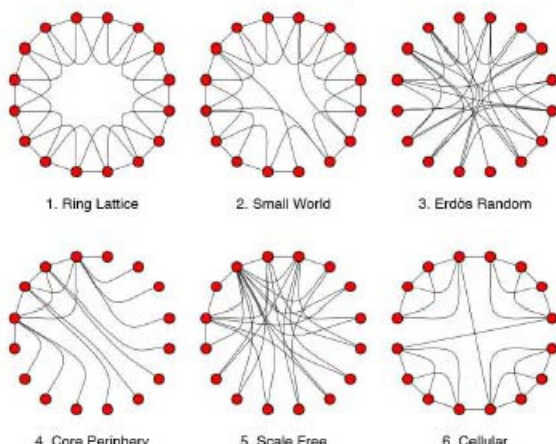
Summary

- Erdos-Reny networks
 - Random network, IID links
- Ring Lattice
 - Circle layout, k-closest neighbors
- Scale free
 - The degree distribution obeys a power law
- Small-world
 - Ring with a few extra hubs
- Cellular
 - Highly connected cells connected by a link to few other cells
- Core periphery
 - Single strong component with high level of peripherals

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Summary



1. Ring Lattice 2. Small World 3. Erdős Random

4. Core Periphery 5. Scale Free 6. Cellular

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